

# A Generalized Fibonacci LSB Data Hiding Technique

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*Abstract – In this contribution, we present a novel embedding scheme based on the generalized Fibonacci decomposition. This representation allows an innovative bit plane decomposition, suitable for embedding purposes. A comparison between the classical Least Significant Bit (LSB) method and the proposed scheme has been performed by considering the perceptual degradation of the mark with respect to the hiding capacity.*

*Keywords: Data hiding, Security, Fibonacci, Image Quality.*

## I. INTRODUCTION

With the increasing availability of multimedia digital data, together with the spreading of Internet, the need is growing for tools to protect and to authenticate the digital information in the distributed, un-secure, and open scenario. Data hiding techniques can be applied to this aim: by introducing ‘ad hoc’ digital information, in an imperceptible and permanent way, into the data to be protected, it is possible to identify the ownership of the data, to detect tampering attempts, and to define the policies in the use of the data.

Several digital data hiding schemes have been proposed in the past; they can be classified depending on the selected embedding domain: spatial [1-3] or transform-based embedding methods such as DCT, Fourier, or wavelet [4-6].

In order to be effective, several constraints should be observed, depending on the application. Invisibility, robustness and security are the most important ones. In fact, every embedding system should not affect the perceptual quality of the video (invisibility) producing noticeable distortions in the received data. Besides, the mark should not be significantly altered by malicious (an attempt to alter the mark) or unintentional (compression, transmission or filtering) operations (robustness). Furthermore, it should not be possible to

remove the mark from the image/video, even if the embedding scheme is known (security).

These constraints can not be achieved at the same time. Increasing the robustness, for example, generally increases the visibility of the mark, and increasing the amount of data to be embedded, usually weakens the security of the system.

In this work, we apply the generalized Fibonacci theory to the data hiding framework by adopting the classical LSB scheme.

The paper is organized as follows: in Section II the embedding performed in the spatial domain is described, while in Section III the Fibonacci sequence and the generalized version of it is presented. The proposed method is described in Section IV. Experimental results are reported in Section V. Finally, in Section VI we draw our conclusions.

## II. SPATIAL EMBEDDING

One of the simplest techniques for embedding digital data into a digital cover is the LSB method [1]. In this scheme, the secret message is inserted into the least-significant bit plane of the cover image either by directly replacing bits or by modifying bits according to a particular ‘inverse’ function. The luminance component of an image of size  $N \times N$  can be decomposed into bit planes through the binary representation.

The characteristic of this representation is that it is not redundant. This means that the binary decomposition of an integer is unique. Considering a gray-scale image (8 bpp), each pixel can assume a value in the interval  $[0, 255]$ . This value  $D$ , in the binary system is equivalent to an 8 bit representation:

$$D = \sum_{i=0}^7 b_i \cdot 2^i$$

where  $b \in \{0,1\}$ .

The main advantage of such a technique is that the modifications of the LSB plane do not affect the

human perception of the overall image quality as the amplitude variation of the pixel luminance values is bounded by  $\pm 1$ . The drawback of such an easy and high capacity method is its weakness. Any luminance modification can severely affect the watermark detection. In order to increase the robustness of LSB embedding schemes, recently new methods operating in different bit-planes or in a mixture of them [1], [7] have been proposed.

### III. THE FIBONACCI DECOMPOSITION

The aim of this work is to investigate a different bit planes decomposition, based on the Fibonacci  $p$ -sequences.

The classical Fibonacci numbers were introduced in the 13th century by Leonardo of Pisa a.k.a. Fibonacci. Being born in about 1170, Leonardo was the most important mathematician of the middle ages. In his book, Liber Abaci [8], he explained the Hindu-Arabic numbers and their use in computations. In this book he introduced the sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

It is defined by the recurrence:

$$F(n) = \begin{cases} 0 & n < 0 \\ 1 & n = 0 \\ F(n-1) + F(n-2) & n > 0 \end{cases} \quad (1.1)$$

This sequence is a particular case of a larger family of sequences. In the general case, each element is given from the sum of two previous elements which are not necessarily consecutive [11]. The most common generalization of Fibonacci numbers is the Fibonacci  $p$ -numbers sequence defined as follows:

$$F(n) = \begin{cases} 0 & n < 0 \\ 1 & n = 0 \\ F_p(n-1) + F_p(n-p-1) & n > 0 \end{cases} \quad (1.2)$$

in which each element depends on the previous one and on the  $p$ -th previous element of the sequence. As in the classical case, the elements of the Fibonacci  $p$ -sequences can be used to represent a positive natural number. The number  $D$  can be converted as follows:

$$D = \sum_{i=p}^{n-1} a_i \cdot F_i(i)$$

where  $p \geq 0$ . The sequence of coefficients:

$$a = \{a_{n-1}, \dots, a_p\}_p$$

is the Fibonacci  $p$ -code representation of the decimal number  $D$ . Unlike the binary representation, the Fibonacci one is redundant. This means that more than one sequence can represent the same number. It is useful to highlight the differences between the power of two representation and the Fibonacci one. As shown in Table 1, given a defined number of bits, the binary decomposition allows one to represent a larger range of numbers. For example, using 8 bits it is possible to represent the numbers in the interval [0; 255]. With the same amount of bits, using the Fibonacci 1-sequence it is possible to represent the numbers in the range [0; 21].

It is evident that using the same amount of bits the power of two representation allows to cover a larger interval of numbers than the Fibonacci sequence. It can be noticed that when  $p=0$ , we obtain the classical power of 2 sequence, while  $p=1$  results in the classical Fibonacci sequence.

If we apply the decomposition to an image, we obtain more planes than in the traditional LSB method and it's demonstrated that the distortion introduced by changing bit values in the power of two representation is bigger than in the Fibonacci decomposition.

	n	0	1	2	3	4	5	6	7	8	9	10
p												
0		1	2	4	8	16	32	64	128	256	512	1024
1		1	1	2	3	5	8	13	21	34	55	89
2		1	1	1	2	3	4	6	9	13	19	28
3		1	1	1	1	2	3	4	5	7	10	15
4		1	1	1	1	1	2	3	4	5	6	8

Table1: Generalized Fibonacci  $p$ -sequences for different values of  $p$  and  $n$

One drawback presented by the Fibonacci representation is the redundancy. In fact each decimal number can have more than one representation. A unique representation is obtained by applying Zeckendorf theorem [9]: "Each positive integer  $m$  can be represented as the sum of distinct numbers in the sequence of Fibonacci numbers using no two consecutive Fibonacci numbers".

For example, for  $p = 1$  the number 5 can be coded as 1000 or 0110. According to Zeckendorf condition, the codeword 0110 is not valid.

Based on these considerations, the embedding method introduced in the next Section is characterized by a variable embedding capacity, depending on the content of the image, on the selected  $p$ -sequence, and on the watermark to be inserted.

#### IV. THE PROPOSED EMBEDDING SCHEME

Let us define  $I(x, y)$  the cover image, and  $w(x, y)$  the data to be embedded. In our experiment they have the same size. The proposed embedding scheme can be summarized as follows:

1. The cover image  $I(x, y)$  is decomposed into bit planes  $I_p(x, y) = F_p\{I(x, y)\}$  by using the Fibonacci  $p$ -decomposition computed using the specified  $p$ -sequence.
2. The selected plane is considered. For each bit the fulfilment of Zeckendorf condition is checked. If it is verified, the mark is inserted otherwise the following is considered. The simplest method used in our experiment is the substitution of the selected bit value with the corresponding watermark bit. An improved version is based on the following additive scheme:

$$I_p(x, y) = I_p(x, y) + \alpha w(x, y)$$

3. Once the whole mark has been inserted, the image is reconstructed from the bit planes. The watermarked image is therefore recomposed.



Figure 1: from left to right, up to down. Original image, classical LSB watermarked image, LSB Fibonacci watermarking, first significant bit watermarked image.

With respect to this basic scheme, several modifications have been tested to increase the robustness of the system.

Among these, we cite the use of a block-wise embedding scheme in which each block has a different embedding strength value according to some HVS-based features. We are investigating the contrast visibility factor to drive such a scheme.

#### V. EXPERIMENTAL RESULTS

The value of  $\alpha$  represents the strength of the watermark. The greater is  $\alpha$ , more robust is the watermarking system. Unfortunately, it results also in images in which the user can perceive the artefacts. The same considerations are also valid for the bit plane selected for the embedding.

In Figure 1, the original image and different versions of the watermarked image are shown. The upper right one has been obtained by using the classical LSB watermarking scheme. The lower left and right ones are obtained using the proposed method. As can be noticed, the impact of the watermark in the latter image is much stronger than in the third one.

To test the effectiveness of the detection scheme, the detector response to the watermarked image "Lena" to 500 randomly generated marks has been considered. As can be noticed in Figure 2, the correlation peak corresponds to the true mark presentation.

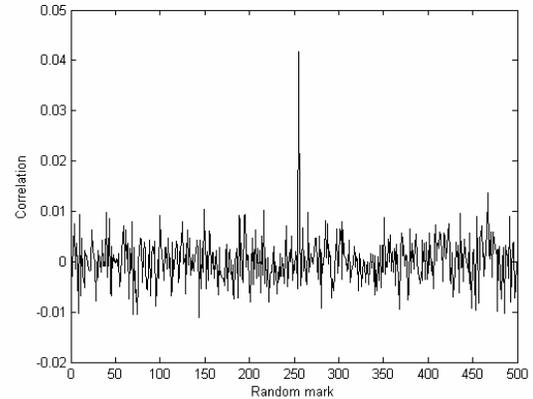


Figure 2: detector response to the watermarked image "Lena" to 500 randomly generated watermarks. Only watermark number 250 matches the embedded one.

To understand the visual impact of the different plane selection, the corresponding PSNR has been computed. As can be noticed in Figures 3 and 4, the

perceptual impact changes with the selected plane. For example, when the embedding is performed in the second bit plane, visual artifacts are introduced.



Figure 3: The watermarked image when  $p= 1$  and selected plane= 2 (a) and selected plane= 11 (b).

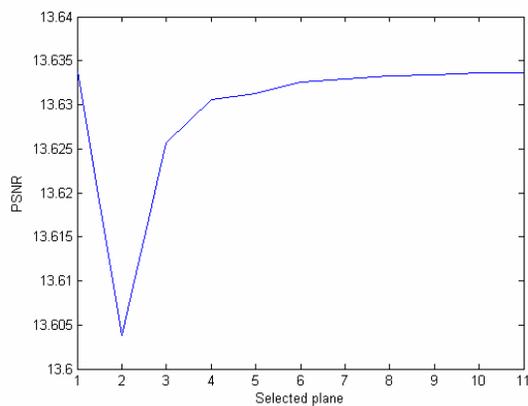


Figure 4: PSNR when  $p= 1$  for different selected plane.

## VI. CONCLUSIONS

In this paper, a new embedding scheme based on the generalized Fibonacci decomposition has been presented. It allows freedom in the selection of the bit planes used for the embedding. The effectiveness of the proposed method has been confirmed by several tests. The results obtained show a good impact also on the perceived quality of the watermarked images.

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